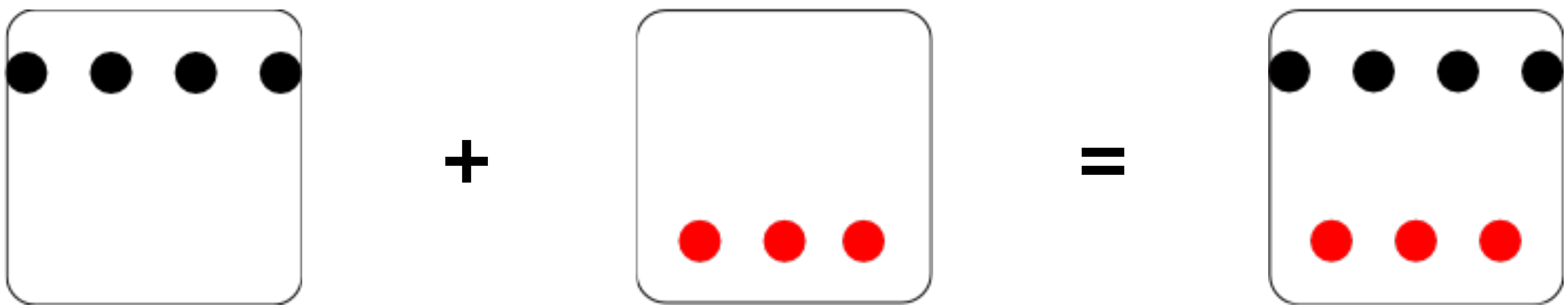
Addition

Definition and Properties

If we combine a set of 4 balls with a set of 3 balls, we obtain a set containing 7 balls.



4 + 3 = 7

An we write

$$4 + 3 = 7 \text{ or } \begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$$

The number 7 is called the **sum** of the numbers 4 and 3. The numbers 4 and 3 are called **addends** or components of the sum.

If we combine two sets of objects into one set, then the number of objects in this set is the sum of the numbers of objects in each of the given sets.

Single Digit Addition

The Decimal Addition Table

The table is grid of 10 rows (horizontal squares labeled 0 - 9), and ten columns (vertical squares labeled 0 - 9). The upper left square show the operation symbol, in this case the plus symbol +. The tool is used to lear addition of single digit numbers. To fin the sum of 3 + 5, you go in the first row and find number 3, then go down the column until you find column 5. The intersection will show you the sum 8.

Decimal Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	11	11	12	13	14	15	16
8	8	9	10	12	12	13	14	15	16	17
9	9	10	11	13	13	14	15	16	17	18

3 + 5 = 8

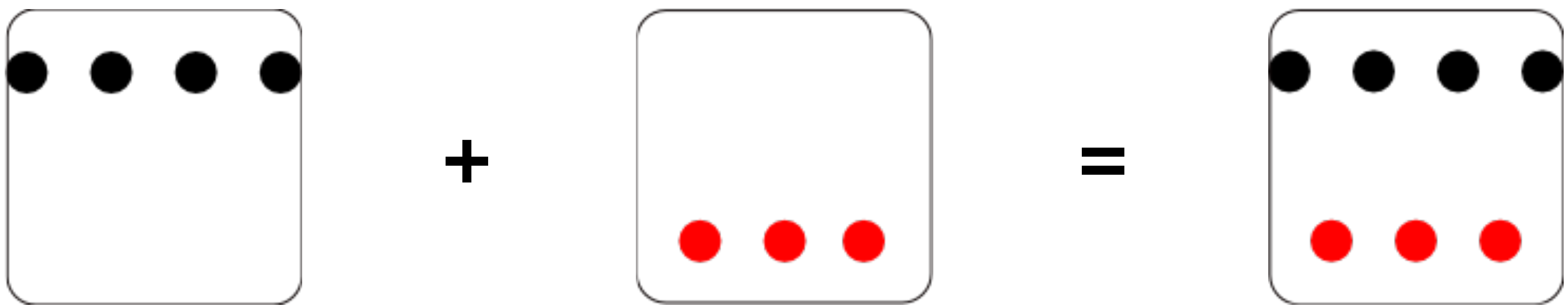
5 + 3 = 8

Whole Numbers Addition Identity

The sum of two numbers, one of which is zero, equals the other number. For example:
1 + 0 = 1, 7 + 0 = 7. Number zero is called the addition ***identity***.

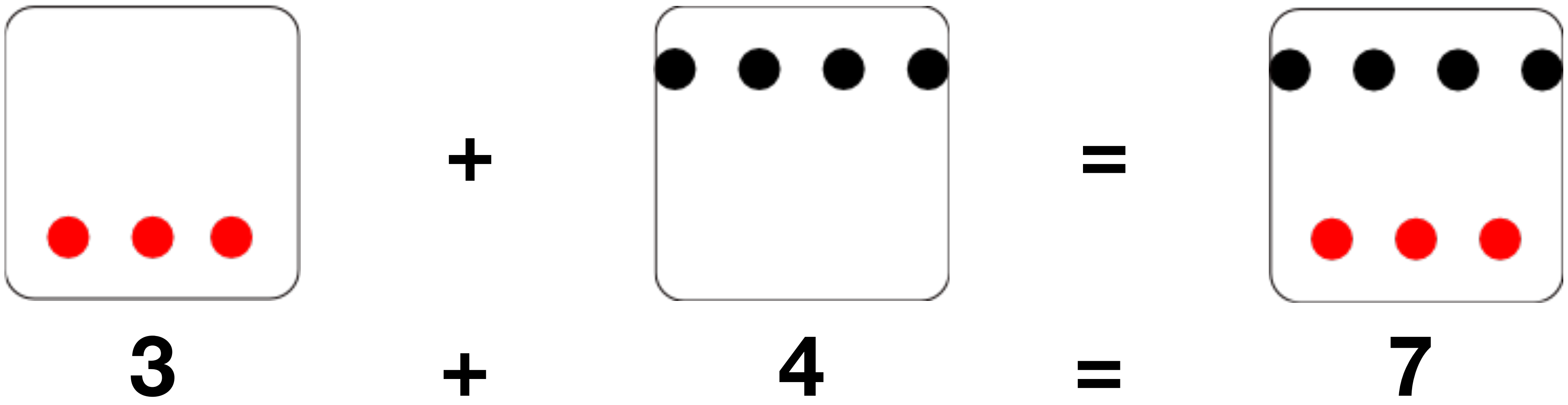
Whole Numbers Addition Commutative Law

The sum 4 + 3 means the number of marbles we will get if we throw 4 marbles into the bag, then 3.



4 + 3 = 7

If we threw these balls into the bag in a different order, e.g. first 3, then 4, we would get the same number of balls in the bag as before.



We can then write 4 + 3 = 3 + 4.

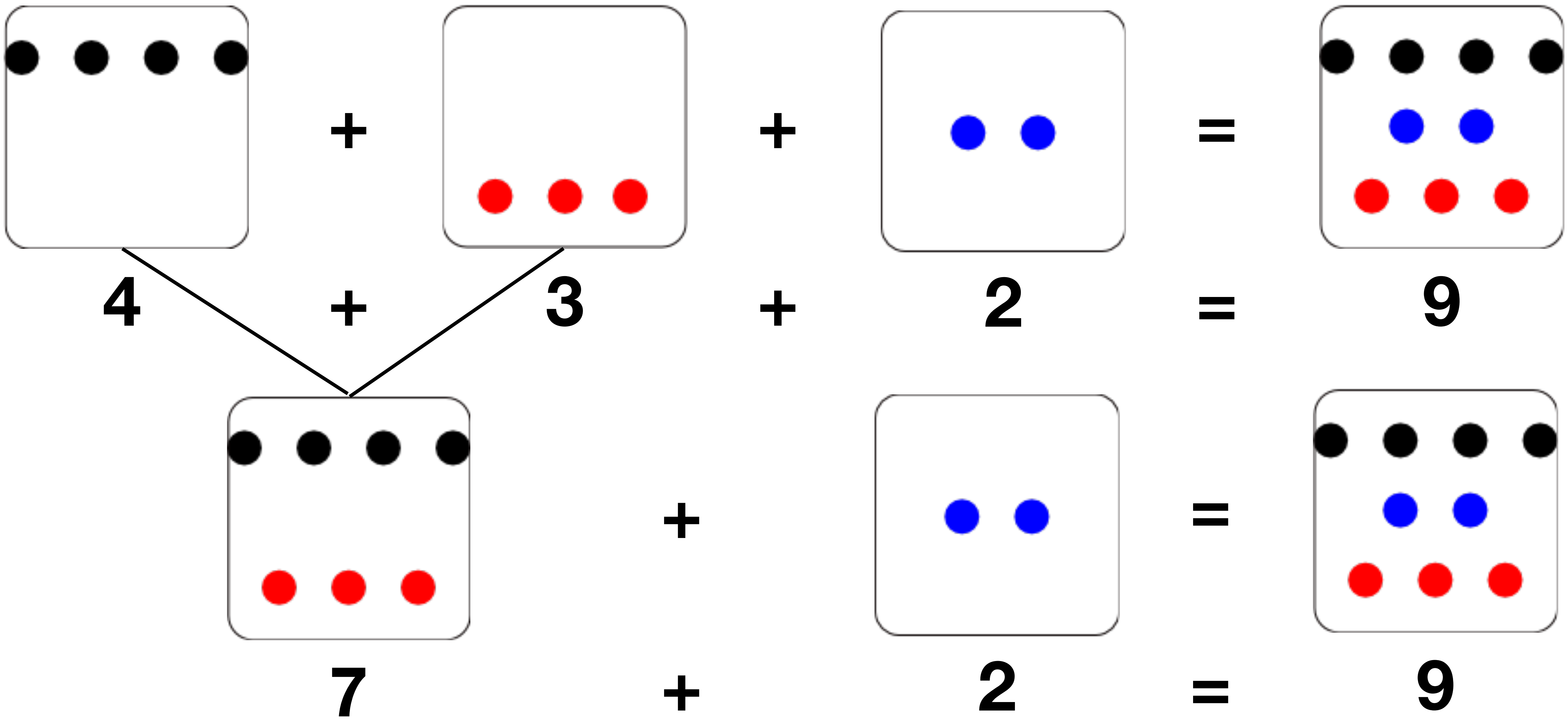
In general, for all whole numbers a, b
a + b = b +a

It follows that:
the sum of several numbers does not depend on the order in which we add them.

Whole Numbers Addition Associative Law

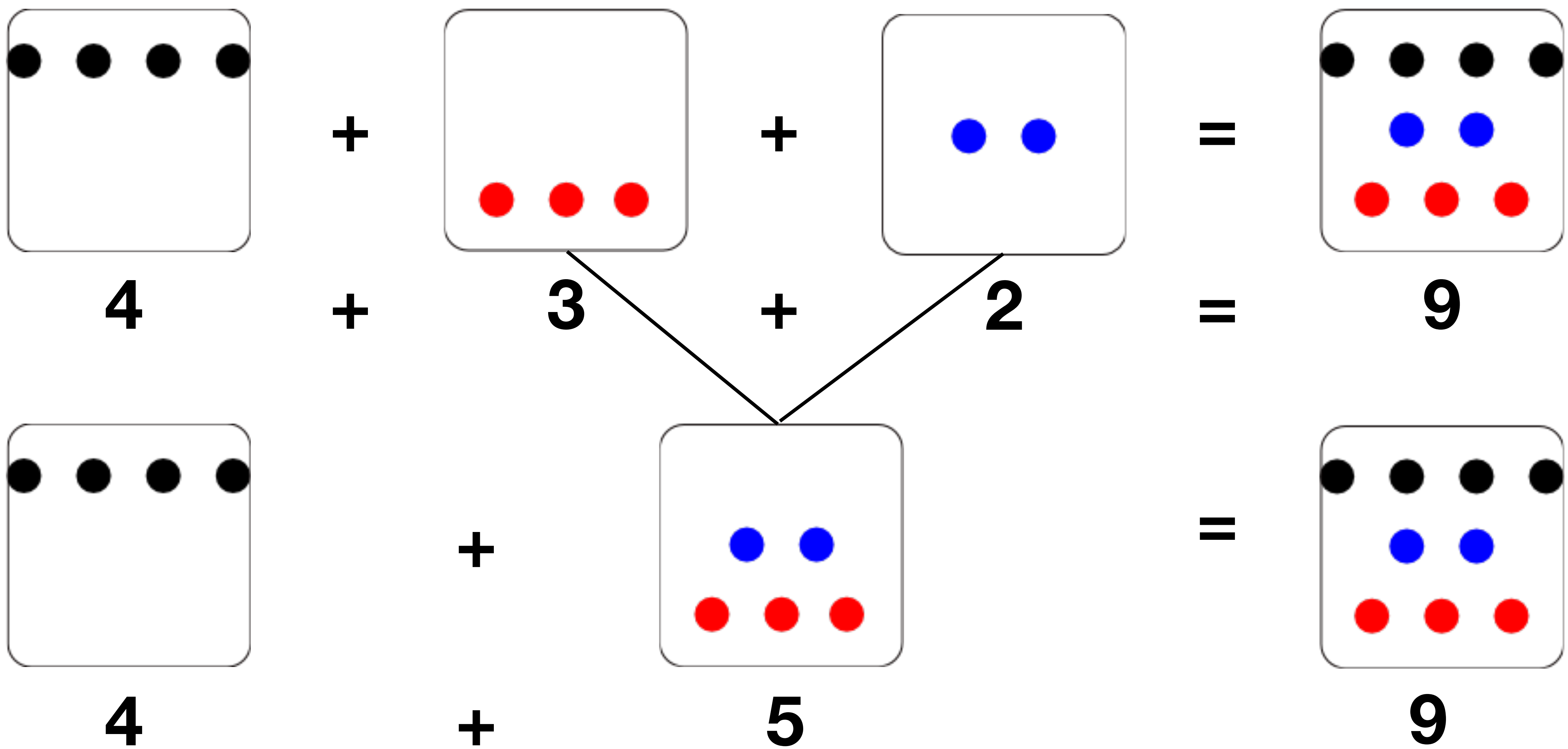
Three packages weighing 4 kg, 3 kg, and 2 kg have a combined weight of: 4 kg + 3 kg + 2 kg.

If two of these packages, e.g., 4 kg and 3 kg, are packed into one package, we will receive two packages weighing 7 kg, and 2 kg, which have a combined weight of: 7 kg + 2 kg.



we can say that: 4 +3 + 2 = 7 + 2

Similarly, by replacing the 3 kg and 2 kg parcels with an 5 kg parcel, we see that:



we can say that: 4 +3 + 2 = 4 + 5

We see, therefore, that:
The sum will not change if we replace some addends
by their sum.
We call this property of sums as the associative law.

To indicate which addends we should replace with their sum, we use parentheses.

Thus, the expression: (4 + 3) + 2 means 7+ 2.
Similarly, 4 + (3 + 2) means 4+ 5.

Therefore, to calculate a sum that includes parentheses,
we replace each parentheses with the number we obtain by performing the operations indicated in that parentheses.

In general, for all whole numbers a, b, c

(a + b) +c = a + (b+ c)

Two Digits Decimal Numbers Addition

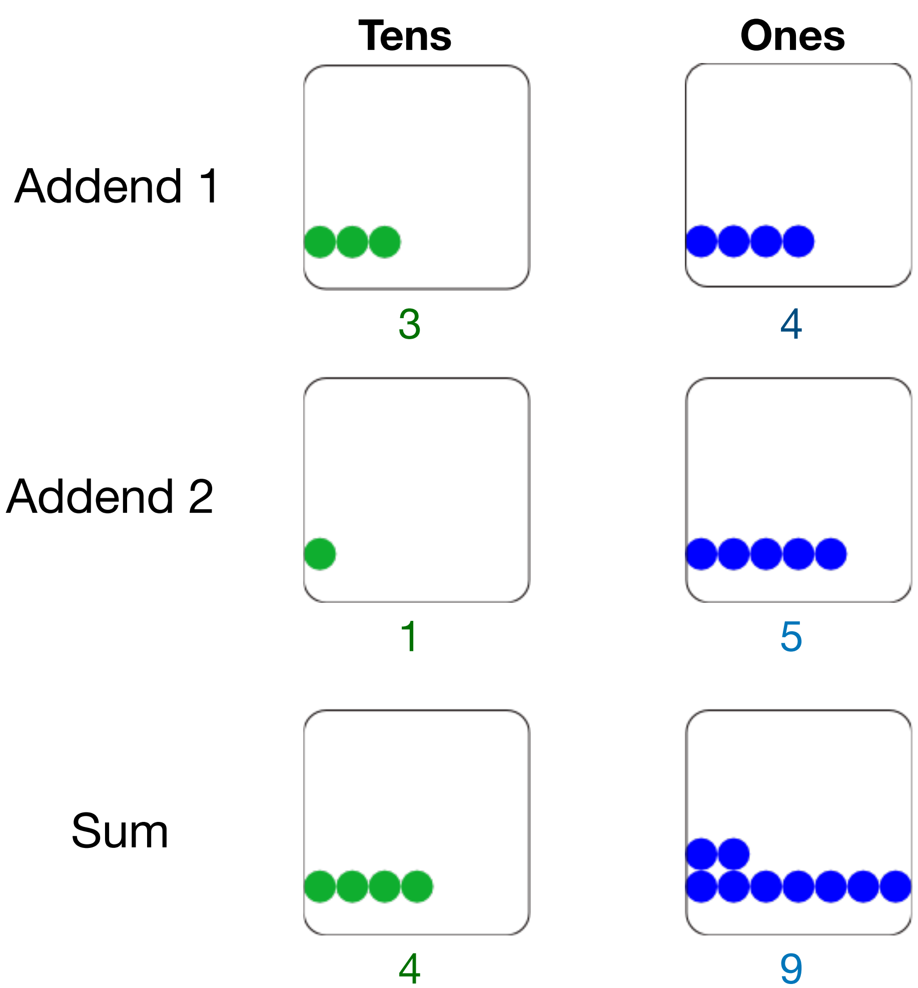
Case I - No Regrouping (No Carrying)

We have a number, for example, 35. This number represents how many marbles we would get if we took 3, 10, and 5 marbles. So we can write:

$$34 = 30 + 4.$$

Similarly

$$15 = 10 + 3.$$



In long form notation:

$$\begin{array}{r} 34 \\ +15 \\ \hline 49 \end{array}$$

Remember that the amount of ones, tens can not be larger than 10. If we were to have ten ones, that should represent a ten and would need to be on the tens basket.

In case I, notice that the addition of ones (zeroth order) digits, and tens (first order) digits is less than 10, so there is no need to regroup.

Two Digits Decimal Numbers Addition

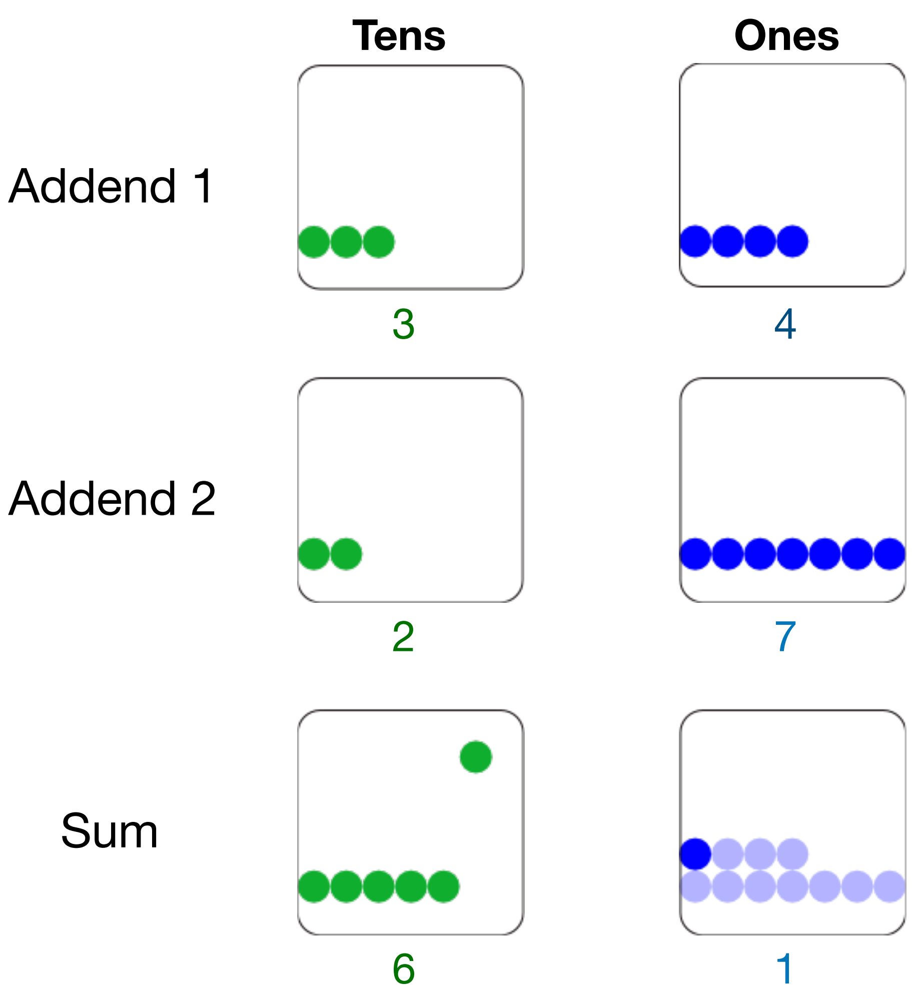
Case II - With Regrouping (with carrying)

We have a number, for example, 35. This number represents how many marbles we would get if we took 3, 10, and 5 marbles. So we can write:

34 = 30 + 4.

Similarly

27 = 10 + 3.



In long form notation:

carry	1	
	3	4
+	2	7
	6	1
		(11)

Remember that the amount of ones, tens can not be larger than 10. If we were to have ten ones, that should represent a ten and would need to be on the tens basket.

In case II, notice that the addition of ones digits 11 = 10 + 1 (larger than ten), so we have to regroup ten ones and carry it to the tens. In the ones we are left wth one one.

Multiple Digits Decimal Numbers Addition

General Case

We need to add several numbers, for example, 1457, 683, and 2795.

Let's write all the additions below each other so that the units' digits are in one column, as well as the tens', hundreds', and thousands digits, and underline them.

So, in our case, we'll write in long form notation:

carry	1	2	1			
	1	4	5	7		Let's add ones: 7 + 3 + 5 =15. 1 ten and 5 ones
		6	8	3		Let's add tens: 1 + 5 + 8 + 9 = 23. 2 hundreds and 3 tens
+	2	7	9	5		Let's add hundreds: 2 + 4 + 6 + 7 = 19. 1 thousand and 9 hundreds
	4	9	3	5		Let's add thousands: 1 + 1 + 2 = 4. 4 thousands.